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CONTROL OF THE MOTION OF A SOLID ROTATING ABOUT ITS CENTRE OF MASS*

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Problems of controlling the spherical motion of a rotating solid when the controlling torques delivered to the body by the controls do not contain an x-component and their axes are not the principal central axes of inertia of the body are investigated. It is shown that as the transverse components of the angular velocity vector are suppressed and the orientation of the non-principal axis of inertia of the body stabilizes, there is an accompanying drop in the angular velocity of twist and, in the final analysis, the rotating body loses its gyroscopic properties. On the other hand, control of the uniform rotation of the body about a principal axis of inertia and of its orientation in inertia space excludes a marked dynamical effect. Control algorithms are porposed to guarantee stability of the relevant motions of the body when the control parameters are subject to constraints. The efficiency of these solutions is confirmed by modelling experiments.

1. Statement of the problem. We introduce three right-handed Cartesian coordinate systems, all with origin O at the centre of mass of the solid: a rigidly attached system xyz, whose axes do not coincide with the system $x_*y_*z_*$ of the principal central axes of inertia of the body, and an inertial coordinate system XYZ.

The relative positions of the xyz and $x_*y_*z_*$ bases are characterized by angles ϑ, ψ and φ (Fig.1). The representation r of a vector R in the xyz basis is expressed in terms of its representation $\mathbf{r}_*$ in the $x_*y_*z_*$ basis by the formula

$$\mathbf{r} = B\mathbf{r}_*,\ B = \{\beta_{ij}\}\ (i, j = 1, 2, 3)$$

(B is the matrix of direction cosines).

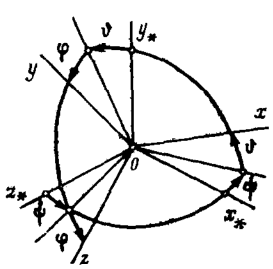

Fig.1

Describing the rotary motion of the solid body in the xyz basis by the dynamical Euler equations

$$J\omega^{\cdot} + \omega \times J\omega = \mathbf{M},\ \omega = \{\omega_x, \omega_y, \omega_z\} \tag{1.1}$$

we note that the inertia matrix J is related to the inertia matrix $J_* = \operatorname{diag}\{J_1, J_2, J_3\}$ in the $x_*y_*z_*$ basis by the expression

$$J = BJ_*B'$$

To fix our ideas, we shall assume that $J_1 < J_2 \leqslant J_3$ (the prime in the formula denotes transposition).

It is assumed that the motion of the body is observed in the rigidly attached coordinate system xyz; the controlling torque $\mathbf{M}$ in that system has the following structure: $\mathbf{M} = \{0, M_y, M_z\}$.

Let ξ and η denote fixed unit vectors in the xyz basis and the XYZ inertial space, respectively. The motion of η relative to the rigidly attached coordinate system is governed by the equation

$$\eta^{\cdot} + \omega \times \eta = 0 \tag{1.2}$$

Assume that an angular velocity $\omega_x = \Omega\ (|\omega_y|, |\omega_z| \ll \Omega)$ is imported to the body.

In view of the special structure of the controlling torque $\mathbf{M}$, the effect of the control

Prikl.Matem.Mekhan., 54, 1, 18-25, 1990 process on the spherical motion of the body exhibits several singular features.

Since the x-component of $\mathbf{M}$ is zero, it is impossible to stabilize uniform motion of the body about the x axis relative to the xyz basis:

$$\omega^0 = \{\Omega,\ 0,\ 0\},\ \Omega = \text{const} \tag{1.3}$$

Thus, the suppression of the angular velocities ω_y and ω_z by controlling torques

$$M_y = k\omega_y,\ M_z = k\omega_z,\ k < 0 \tag{1.4}$$

causes a drop in the angular velocity ω_x and ultimately cancels out the gyroscopic properties of the rotating body.

The explanation for this situation is that the vector (1.3) is not an equilibrium position of system (1.1). The control (1.4) for its part, guarantees asymptotic stability of the zero solution of system (1.1). This may be verified by using the Lyapunov function $2V = \omega' J\omega$ and relying on the Barbashin-Krasovskii Theorem /1/.

The situation is analogous when it is necessary to control the uniaxial orientation of the rotating body, with the fixed unit vector ξ in the xyz system taken to be $\xi = \{1, 0, 0\}$ and the controlling torque defined as

$$M_y = \mu\eta_3 + k\omega_y,\ M_z = -\mu\eta_2 + k\omega_z, \quad \mu > 0,\ k < 0 \tag{1.5}$$

The control (1.5) coincides with the y- and z-components of the controlling torque $\mathbf{M} = \mu\xi \times \eta + k\omega$ which solves the problem of uniaxial orientation of the solid body when the component M_x of the torque $\mathbf{M}$ does not vanish /2, 3/.

These features of the control processes are illustrated in Figs.2 and 3, in which curves *1-3* show the variation in time of the angular velocities ω_x, ω_y and ω_z $(\omega_x(0) = \Omega = 1\ \text{s}^{-1})$. Curve *4* in Fig.3 shows the variation during the control process of the cosine of the angle between the x axis (the vector ξ) and the target direction (the vector η). The graphs were drawn for a body with an ellipsoid of inertia having the following parameters:

$$J_1 = 0.1 \cdot 10^n,\ J_2 = 10^n,\ J_3 = 1.2 \cdot 10^n\ (\text{kgm}^2);\ \vartheta = 5°,\ \psi = 0,\ \varphi = 45°$$

In the algorithm (1.4) $k = -0.3 \times 10^n$ Nms, while in (1.5) $k = -0.313 \times 10^n$ Nms, and $\mu = 0.05 \times 10^n$ Nm. For $\vartheta = 1°$ and the same values of the angles ψ and φ one observes a less intense drop in the value of ω_x ($\sim$ by 4% after 40 seconds of motion).

The specific features of the structure of the controlling torque dictate a different approach to control problems involving rotating solids.

Note that rotation of the body at an angular velocity

$$\omega = \omega_* = \Omega_*\xi,\ \xi = \{\beta_{11}, \beta_{21}, \beta_{31}\},\ \ \Omega_* = \text{const} \tag{1.6}$$

is an equilibrium position of system (1.1) (corresponding in the $x_*y_*z_*$ basis to permanent rotation about the x_* axis at an angular velocity of Ω_*).

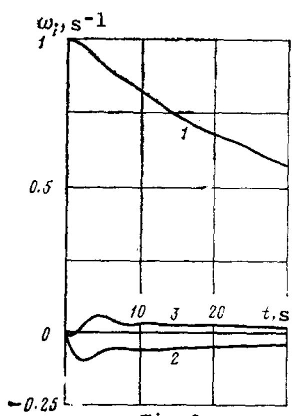

Fig.2

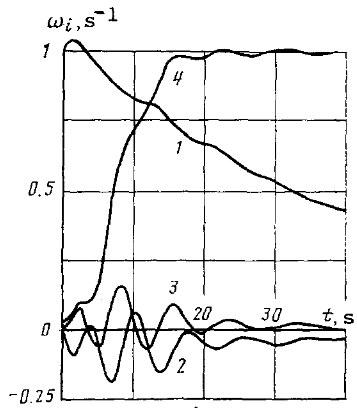

Fig.3

If the mismatch between the x and x_* axes of the xyz and $x_*y_*z_*$ bases is small (the angle φ may take arbitrary values), the rotation of the body at angular velocity (1.6) differs only slightly from the motion (1.3). Therefore, the torque $\mathbf{M} = \{0, M_y, M_z\}$ should be used to stabilize the rotation (1.6). But in the control of uniaxial orientation, the fixed unit vector ξ in the xyz basis should be taken as $\xi = \{\beta_{11}, \beta_{21}, \beta_{31}\}$ (the unit vector of the principal central axis of inertia x_* in the xyz basis).

We now formulate two control problems.

Problem 1. On the basis of the information available in the xyz coordinate system about the angular velocity vector ω of the rotating body and the unit vector ξ, it is required to synthesize a control $\mathbf{M} = \{0, M_y, M_z\}$ under which the permanent rotation (1.6) of the body is stable.

Problem 2. Assuming that the vectors $\boldsymbol{\omega}$ and $\boldsymbol{\eta}$ are measurable in the xyz system, it is required to synthesize a controlling torque $\mathbf{M} = \{0, M_y, M_z\}$ under which the uniaxial orientation of the body is stable:

$$\boldsymbol{\xi} = \boldsymbol{\eta}, \quad \boldsymbol{\omega} = \boldsymbol{\omega}_* \tag{1.7}$$

2. ***Controlling rotary motion.*** Using the formula

$$\boldsymbol{\omega} = \mathbf{X} + \Omega_*\boldsymbol{\xi}, \quad \mathbf{X} = \{X_i\} \quad (i = 1, 2, 3)$$

we introduce now variable X_i and, using (1.1), write the equation of perturbed motion of the body as follows:

$$J\mathbf{X}^{\cdot} + \Omega_*\boldsymbol{\xi} \times J\mathbf{X} + \mathbf{X} \times J(\Omega_*\boldsymbol{\xi} + \mathbf{X}) = \mathbf{M} \tag{2.1}$$

The solution of Problem 1 will be a torque $\mathbf{M}$, application of which makes the trivial solution of Eq.(2.1) stable.

Choosing the controlling torque as

$$M_y = k(X_2 - \beta_{21}(\boldsymbol{\xi}'\mathbf{X})), \quad M_z = k(X_3 - \beta_{31}(\boldsymbol{\xi}'\mathbf{X})) \tag{2.2}$$

or, in matrix notation,

$$\mathbf{M} = kQ\mathbf{X} \tag{2.3}$$

$$Q = \begin{Vmatrix} 0 & 0 & 0 \\ -\beta_{11}\beta_{21} & 1-\beta_{21}^2 & -\beta_{21}\beta_{31} \\ -\beta_{11}\beta_{31} & -\beta_{21}\beta_{31} & 1-\beta_{31}^2 \end{Vmatrix}$$

we shall show that the control (2.2) is the desired solution.

The equation of the perturbed motion (2.1), rewritten in terms of the $x_*y_*z_*$ basis, is

$$J_*\mathbf{x}^{\cdot} + \Omega_*\boldsymbol{\xi}_* \times J_*\mathbf{x} + \mathbf{x} \times J_*(\Omega_*\boldsymbol{\xi}_* + \mathbf{x}) = \mathbf{M}_* \tag{2.4}$$

$$\boldsymbol{\xi}_* = \{1, 0, 0\} = B'\boldsymbol{\xi}, \quad \mathbf{x} = \{x_i\} = B'\mathbf{X}$$

and by (2.3) the controlling torque $\mathbf{M}_*$ is defined by the relation

$$\mathbf{M}_* = kQ_*\mathbf{x} \tag{2.5}$$

$$Q_* = B'QB = \begin{Vmatrix} 0 & -\beta_{11}\beta_{12} & -\beta_{11}\beta_{13} \\ 0 & 1-\beta_{12}^2 & -\beta_{12}\beta_{13} \\ 0 & -\beta_{12}\beta_{13} & 1-\beta_{13}^2 \end{Vmatrix}$$

We now consider a positive definite function of x_2 and x_3

$$2V_1 = J_2(J_2 - J_1)x_2^2 + J_3(J_3 - J_1)x_3^2 \tag{2.6}$$

which is a first integral of system (2.4) if $\mathbf{M}_* = 0$ /4/.

The derivative of V_1 with respect to time along trajectories of Eqs.(2.4) and (2.5)

$$V_1^{\cdot} = \mathbf{y}'S\mathbf{y}, \quad \mathbf{y} = \{x_2, x_3\}$$

$$S = k\begin{Vmatrix} (J_2 - J_1)(1-\beta_{12}^2) & \tfrac{1}{2}(2J_1 - J_2 - J_3)\beta_{12}\beta_{13} \\ \tfrac{1}{2}(2J_1 - J_2 - J_3)\beta_{12}\beta_{13} & (J_3 - J_1)(1-\beta_{13}^2) \end{Vmatrix}$$

where it is assumed that

$$k < 0, \quad (J_2 - J_1)(J_3 - J_1)\beta_{11}^2 - \tfrac{1}{4}(J_3 - J_2)^2\beta_{12}^2\beta_{13}^2 > 0 \tag{2.7}$$

is a negative definite quadratic form with respect to the vector $\mathbf{y}$. This implies asymptotic stability with respect to the variables x_2 and x_3.

It follows from an analysis of the first equation of system (2.4) and the torque M_1 in (2.5) that, as the variables x_2 and x_3 approach zero asymptotically, the variable x_1 tends to $c = \text{const}$.

Thus, if conditions (2.7) are satisfied, the trivial solution of Eq.(2.4) is asymptotically $\mathbf{y}$-stable (in the terminology of /5/). This implies that the trivial solution of system (2.1) is stable.

To sum up the control (2.2), which is conveniently written as

$$M_y = k(\omega_y - \beta_{21}(\boldsymbol{\xi}'\boldsymbol{\omega})), \quad M_z = k(\omega_z - \beta_{31}(\boldsymbol{\xi}'\boldsymbol{\omega})) \tag{2.8}$$

is a solution of Problem 1.

3. Control of uniaxial orientation. Rewrite Eqs.(1.1) and (1.2) as

$$JX^{\cdot} = M - \Omega_* \xi \times JX - X \times J(X + \Omega_* \xi)$$
$$Z^{\cdot} = Z \times (X + \Omega_* \xi) + \xi \times X \tag{3.1}$$
$$X = \omega - \Omega_* \xi, \quad Z = \eta - \xi$$

If the controlling torque **M** had an x-component, a control of the form, say /2, 3/,

$$M = \mu \xi \times Z + KX, \ \mu > 0, \ K = \operatorname{diag}\{k_1, k_2, k_3\} \tag{3.2}$$

with $k_i < 0, k_2 k_3 - {}^1/_4 \Omega_*^2 (J_3 - J_2)^2 > 0 \ (i = 1, 2, 3)$, would guarantee asymptotic stability of the regime (1.7) of uniaxial orientation of the body.

Defining the controlling torque **M** as the sum of the first term of (3.2) without its x-component, and the torque (2.3), we obtain a control

$$M = \mu P Z + kQX, \ \mu > 0$$
$$P = \begin{Vmatrix} 0 & 0 & 0 \\ \beta_{31} & 0 & -\beta_{11} \\ -\beta_{21} & \beta_{11} & 0 \end{Vmatrix} \tag{3.3}$$

which, we claim, is the desired solution.

To prove this, we write Eqs.(3.1) of the perturbed motion in the $x_* y_* z_*$ basis

$$J_1 x_1^{\cdot} = (J_2 - J_3) x_2 x_3 + M_1 \tag{3.4}$$
$$J_2 x_2^{\cdot} = (J_3 - J_1)(x_1 + \Omega_*) x_3 + M_2$$
$$J_3 x_3^{\cdot} = (J_1 - J_2)(x_1 + \Omega_*) x_2 + M_3$$
$$z_1^{\cdot} = x_3 z_2 - x_2 z_3, \ z_2^{\cdot} = (x_1 + \Omega_*) z_3 - x_3 (z_1 + 1)$$
$$z_3^{\cdot} = x_2 (z_1 + 1) - (x_1 + \Omega_*) z_2$$
$$M_* = \{M_i\} = \mu P_* z + kQ_* x, \ z = \{z_i\} = B'Z$$
$$(i = 1, 2, 3)$$
$$P_* = B'PB = \begin{Vmatrix} 0 & -\beta_{11}\beta_{13} & \beta_{11}\beta_{12} \\ 0 & -\beta_{12}\beta_{13} & -(1 - \beta_{12}^2) \\ 0 & 1 - \beta_{13}^2 & \beta_{12}\beta_{13} \end{Vmatrix}$$

and introduce new variables

$$Y_1 = kx_2 - \mu z_3, \ Y_2 = kx_3 + \mu z_2, \ Y_{i+2} = z_i \quad (i = 1, 2, 3) \tag{3.5}$$

In terms of these new variables and the resulting expression

$$M_* = \begin{Vmatrix} -\beta_{11}\beta_{12} Y_1 - \beta_{11}\beta_{13} Y_2 \\ (1 - \beta_{12}^2) Y_1 - \beta_{12}\beta_{13} Y_2 \\ -\beta_{12}\beta_{13} Y_1 + (1 - \beta_{13}^2) Y_2 \end{Vmatrix}$$

for the controlling torque $M_* = \{M_i\}$, the system of Eqs.(3.4) becomes

$$J_1 x_1^{\cdot} = -\beta_{11}\beta_{12} Y_1 - \beta_{11}\beta_{13} Y_2 + k^{-2} (J_2 - J_3)(Y_1 + \mu Y_5)(Y_2 - \mu Y_4) \tag{3.6}$$
$$J_2 Y_1^{\cdot} = \alpha_{11} Y_1 + \alpha_{12} Y_2 + \alpha_{13} Y_4 + \alpha_{14} Y_5 + f_1(Y)$$
$$J_3 Y_2^{\cdot} = \alpha_{21} Y_1 + \alpha_{22} Y_2 + \alpha_{23} Y_4 + \alpha_{24} Y_5 + f_2(Y)$$
$$Y_3^{\cdot} = k^{-1} (Y_2 Y_4 - Y_1 Y_5) - \mu k^{-1} (Y_4^2 + Y_5^2)$$
$$Y_4^{\cdot} = -k^{-1} Y_2 + \mu k^{-1} Y_4 + (\Omega_* + x_1) Y_5 - k^{-1} (Y_2 - \mu Y_4) Y_3$$
$$Y_5^{\cdot} = k^{-1} Y_1 - (\Omega_* + x_1) Y_4 + \mu k^{-1} Y_5 + k^{-1} (Y_1 + \mu Y_5) Y_3$$
$$\alpha_{11} = k (1 - \beta_{12}^2) - J_2 \mu k^{-1}, \quad \alpha_{12} = (J_3 - J_1)(\Omega_* + x_1) - k\beta_{12}\beta_{13}$$
$$\alpha_{13} = (J_1 + J_2 - J_3)(\Omega_* + x_1)\mu, \quad \alpha_{14} = -J_2 \mu^2 k^{-1}$$
$$\alpha_{21} = (J_1 - J_2)(\Omega_* + x_1) - k\beta_{12}\beta_{13}, \quad \alpha_{22} = k (1 - \beta_{13}^2) - J_3 \mu k^{-1}$$
$$\alpha_{23} = J_3 \mu^2 k^{-1}, \quad \alpha_{24} = (J_1 - J_2 + J_3)(\Omega_* + x_1)\mu$$
$$f_1(Y) = -J_2 \mu k^{-1} (Y_1 + \mu Y_5) Y_3, \quad f_2(Y) = -J_3 \mu k^{-1} (Y_2 - \mu Y_4) Y_3$$
$$Y = \{Y_j\} \quad (j = 1, 2, \ldots, 5)$$

Consider the following positive definite function of the vector **Y**,

$$2V_2 = J_2 Y_1^2 + J_3 Y_2^2 + \nu (Y_3^2 + Y_4^2 + Y_5^2), \quad \nu > 0 \tag{3.7}$$

whose time derivative along trajectories of Eqs.(3.6) may be expressed as

$$V_2^{\cdot} = \zeta' S \zeta + R(\mathbf{Y}), \quad S = {}^1/_2 (T + T') \tag{3.8}$$
$$\zeta = \{Y_1, Y_2, Y_4, Y_5\}, \quad R(\mathbf{Y}) = Y_1 f_1(\mathbf{Y}) + Y_2 f_2(\mathbf{Y})$$
$$T = \begin{Vmatrix} \alpha_{11} & \alpha_{12}+\alpha_{21} & \alpha_{13} & \alpha_{14}+\nu k^{-1} \\ 0 & \alpha_{22} & \alpha_{23}-\nu k^{-1} & \alpha_{24} \\ 0 & 0 & \mu\nu k^{-1} & 0 \\ 0 & 0 & 0 & \mu\nu k^{-1} \end{Vmatrix}$$

Since the positive (or negative) definiteness of an analytic function depends entirely on the lowest-order terms in its expansion /4/, it follows that the right-hand side of (3.8), as a function of the vector ζ, will be negative definite if this is true of the quadratic form $\zeta' S \zeta$.

Choose the coefficients μ, ν and k in such a way that the function $\zeta' S \zeta$ is negative definite. Then the first term in (3.8), as a function of $\mathbf{Y}$, will be negative semidefinite, since, in view of the first integral

$$Y_3^2 + Y_4^2 + Y_5^2 + 2Y_3 = 0$$

of system (3.6), it vanishes not only at $\mathbf{Y} = 0$ but also at the point

$$N = \{\mathbf{Y}: \zeta = 0, \ Y_3 = -2\}$$

Analysis of the equations of the first approximation relative to the point N indicates that N is an unstable equilibrium position of system (3.6).

Thus, if $S < 0$ the vector $\mathbf{Y}$ asymptotically approaches zero. Simultaneously, according to the structure of the first equation of system (3.6) and the torque M_1, the coordinate x_1 approaches $c_1 = \text{const}$. The trivial solution of system (3.6) is asymptotically $\mathbf{Y}$-stable implying that the equilibrium position $\mathbf{X} = 0, \mathbf{Z} = 0$ of system (3.1) is asymptotically $\mathbf{Z}$-stable.

The control (3.3), whose components M_y and M_z can be written

$$M_y = \mu(\beta_{31}\eta_1 - \beta_{11}\eta_3) + k(\omega_y - \beta_{21}(\xi'\omega)) \tag{3.9}$$
$$M_z = \mu(\beta_{11}\eta_2 - \beta_{21}\eta_1) + k(\omega_z - \beta_{31}(\xi'\omega))$$

maintains the required orientation of the axis of the body and is thus a solution of Problem 2.

4. ***Allowance for constraints on the control parameters.*** We now extend the results of Sects.2 and 3 to the case in which the torques M_y and M_z, which are related to the control parameters u_1 and u_2 by the formulae

$$M_y = m_y u_1, \quad M_z = m_y u_2$$

$(m_y, m_z > 0$ are constant coefficients), are subject to a constraint

$$\mathbf{M} \in G = \{\mathbf{M}: |u_i| \leqslant 1, \ i = 1, 2\}$$

To that end we shall use the approach proposed in /6/. We first find the maximum M° of the controlling torque $\mathbf{M}$ in the $\mathbf{n}$-direction in the yz plane, where $\mathbf{n} = \{\alpha, \beta\}$ is a unit vector, and determine the relevant values u_1° and u_2° of the control parameters.

If $|\alpha| \geqslant m_y m_z^{-1} |\beta|$, then

$$M^\circ = \frac{m_y}{|\alpha|}, \quad u_1^\circ = \frac{\alpha}{|\alpha|}, \quad u_2^\circ = \frac{m_y}{m_z}\frac{\beta}{|\alpha|}$$

If $|\alpha| < m_y m_z^{-1} |\beta|$, then

$$M^\circ = \frac{m_z}{|\beta|}, \quad u_1^\circ = \frac{m_z}{m_y}\frac{\alpha}{|\beta|}, \quad u_2^\circ = \frac{\beta}{|\beta|}$$

If the torque (2.8) controlling the rotary motion of the body does not belong to the set G, we redefine the controlling torque as

$$M_y^\circ = m_y u_1^\circ, \quad M_z^\circ = m_z u_2^\circ$$

where u_1° and u_2° are the control parameters corresponding to the maximum M° of the available torque relative to the n-direction with unit vector

$$\mathbf{n} = \{M_y / \|\mathbf{M}\|, \ M_z / \|\mathbf{M}\|\}$$

Note that if $M_y = M_y^\circ, \ M_z = M_z^\circ$,

$$V_1^{\cdot} = \gamma_* \mathbf{y}' S \mathbf{y}, \quad \gamma_* = M^\circ / \|\mathbf{M}\|$$

and if $S < 0$ this guarantees that the system will approach the equilibrium position (1.6).

The algorithm (3.3) controlling the uniaxial orientation of the rotationg body may be expressed as the sum of two terms:

$$\mathbf{M} = \mathbf{M}_1(\mathbf{Z}) + \mathbf{M}_2(\mathbf{X})$$

Assume that, for any orientation of the body, the first term satisfies the condition

$$\mathbf{M}_1(\mathbf{Z}) = \mu P\mathbf{Z} \in G$$

The second component of the torque will be evaluated from the formula

$$\mathbf{M}_2(\mathbf{X}) = \varkappa_* kQ\mathbf{X}$$

where $\varkappa_*$ is the maximum value of the parameter $\varkappa$ in the range $0 < \varkappa \leqslant 1$ such that

$$\mathbf{M} = \mathbf{M}_1(\mathbf{Z}) + \varkappa_* kQ\mathbf{X} \in G$$

Repeating the arguments of Sect.3, it can be shown that with this choice of controls, system (1.1) and (1.2) approaches the equilibrium position (1.7).

5. ***Examples.*** Typical patterns of the variation of the angular velocities ω_x, ω_y, ω_z and the cosine of the angle between ξ and η when algorithms (2.8) and (3.9) are applied to control the uniform rotation of a solid about its principal axis of inertia and uniaxial orientation, respectively, are shown in Figs.4 and 5, respectively (for the notation, see Figs.2 and 3).

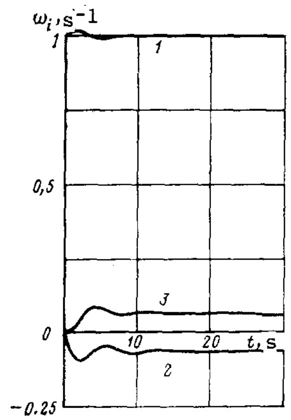

Fig.4

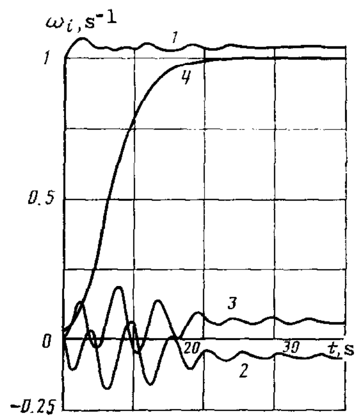

Fig.5

The system was modelled with the parameter values of the inertia ellipsoid specified in Sect.1. To the angles $\vartheta = 5°$, $\varphi = 45°$, $\psi = 0$ there corresponds a unit vector

$$\xi = \{\beta_{i1}\} = \{0.996196, -0.061617, 0.061617\}$$

along the principal axis of inertia x_*. In (2.8), as in the algorithm (1.4), $k = -0.3 \times 10^n$ Nms. In the control of uniaxial orientation the controls are constructed for the following values of the parameters occurring in (3.9): $k = -0.313 \times 10^n$ Nms, $\mu = 0.05 \times 10^n$ Nm.

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